Quiz 4 Polymer Physics Fall 2000
10/18/00

The following equation is a generic form for the differential equation used in class to describe a simple, single mode Debye mechanical relaxation:

$$\tau \frac{d\gamma(t)}{dt} + \gamma(t) = \tau J_U \frac{d\sigma}{dt} + J_R \sigma(t)$$

where σ is the stress, γ is the strain and τ is the time constant.

a) -**Simplify this** equation for a creep measurement,
   -and for a stress relaxation measurement.
   -Give the limits used to solve the differential equation in both cases ($G_U = 1/J_U$, $G_R = 1/J_R$).

b) **-Show that** the solution to the creep differential equation is:

$$J(t) = \frac{\gamma(t)}{\sigma_0} = J_U + (J_R - J_U)\left(1 - \exp\left(\frac{-t}{\tau}\right)\right)$$

   Using for dy/dx + Py = Q, where P and Q are independent of y but can involve x, the solution is y = exp(∫-Pdx) {c + ∫(exp(∫Pdx) Q dx)} where c is a constant that can be solved for the limiting conditions of $J = J_U$ at t=0 or for the t=∞ limit of $\gamma = \Delta J\ \sigma_0$.
   -Sketch J(t) versus t showing J_U and J_R.

c) **-What** is the main assumption involved in a simple single mode Debye relaxation that is the basis of the equation given above.

d) The dynamic modulus, G*, is given by:

$$G^* = G_U - \frac{G_U - G_R}{1 + i\omega\tau}$$

   -From this equation show that:

$$G' = G_R + \frac{(G_U - G_R)\omega^2\tau^2}{1 + \omega^2\tau^2} \text{ and } G'' = \frac{(G_U - G_R)\omega\tau}{1 + \omega^2\tau^2}$$

   -How does this compare with the loss and storage compliance given in class?

e) **-What** are the mechanical analogues for the applied electric field and the electronic displacement in a dielectric relaxation measurement?
   -How do E and D relate to the dielectric constant, ε?
   -How does the polarization, P, relate to D and E?

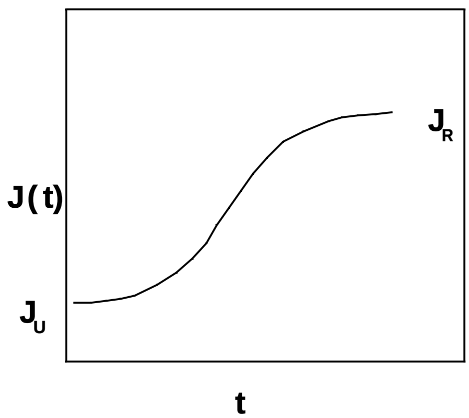
Answers Quiz 4 Polymer Physics

a) Creep: $\tau \frac{d\gamma(t)}{dt} + \gamma(t) = J_R \sigma_0$

J = J_U at t=0 or for the t=∞ limit of $\gamma = (J_U + \Delta J)\,\sigma_0$.

Stress Relaxation: $\gamma_0 = \tau J_U \frac{d\sigma}{dt} + J_R \sigma(t)$

G = $G_U = 1/J_U$ at t=0 or for the t=∞ limit of $\sigma = \Delta G\,\gamma_0$.



b) $\frac{dJ(t)}{dt} + \frac{J(t)}{\tau} = \frac{J_R}{\tau}$

so $P = 1/\tau$ and $Q = J_R/\tau$,

then:

$$J(t) = \exp\left(\frac{-t}{\tau}\right)\left\{C + \frac{J_R}{\tau}\int \exp\left(\frac{t}{\tau}\right)dt\right\} = C\exp\left(\frac{-t}{\tau}\right) + J_R$$

at t = 0 $J(t) = J_U$, so $C = J_U - J_R$. Then,

$$J(t) = J_U \exp\left(\frac{-t}{\tau}\right) - J_R \exp\left(\frac{-t}{\tau}\right) + J_R = J_U + \Delta J\left(1 - \exp\left(\frac{-t}{\tau}\right)\right)$$

c) The main assumption is that the response is proportional to the distance from the relaxed state.

d) The equation is multiplied by (1 - iωτ)/(1 - iωτ) to yield:

$$G^* = G_U - \frac{\Delta G}{1+\omega^2\tau^2} + i\frac{\Delta G \omega\tau}{1+\omega^2\tau^2}$$

which yields the imaginary modulus given. The real modulus given can be obtained by rearrangement of the above equation in the following way,

$$G' = G_U - \frac{\Delta G}{1+\omega^2\tau^2} = \frac{G_U + G_U\omega^2\tau^2 - G_U + G_R}{1+\omega^2\tau^2}$$

$$= \frac{G_R\left(1+\omega^2\tau^2\right) + G_U\omega^2\tau^2 - G_R\omega^2\tau^2}{1+\omega^2\tau^2} = G_R + \frac{\Delta G}{1+\omega^2\tau^2}$$

The loss and storage compliances are given by:

$$J' = J_U + \frac{\left(J_R - J_U\right)}{1+\omega^2\tau^2}$$

$$J'' = \frac{\left(J_R - J_U\right)\omega\tau}{1+\omega^2\tau^2}$$

The τ's in the two equations are not the same but are related.

e) E is analogous to stress and D to strain

D = εE

D = E + 4π P